Routing on Asymptotic Delays in IEEE 802.11 Wireless Ad Hoc Networks

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***Abstract*— We evaluate the delay distribution in IEEE 802.11 wireless ad hoc networks. We show that, under certain assumptions, the asymptotic delay distribution can be expressed as a power law. Using this result, we present a new asymptotic delay based routing protocol.**

I. Introduction

This paper addresses the problem of the evaluation of the delay distribution via analytical means in a multihop wireless network. We assume a wireless network with IEEE 802.11 MAC protocol under the OLSR routing protocol [1]. The routing protocol is a table driven protocol that operates under periodic broadcast control packets.

We denote by W the end to end delay delivery of a packet.

A delay-oriented quality of service for a connection is generally expressed via a maximum acceptable delay T and an overdelay maximum ratio ϵ, requiring that during the connection:

$$P(W > T) < \epsilon$$

Our aim is to present a protocol that evaluates the delay distribution $P(W > T)$ for any large T (when $P(W > T)$ is small) and to use it in order to find a route that satisfies the delay requirement.

In general finding the optimal route that minimizes an overdelay ratio is NP-hard [2]. Nevertheless we will show that the delay distribution at every node router is in power law and that this allows us to specify a polynomial approximation algorithm with error factor $1 + O(T^{-1})$.

II. One hop delay estimate

A. Methodology overview

A wireless node can be seen as a buffer filled by incoming messages and with a single server that performs the CSMA-CA multiple access protocol.

We model this system as an M/G/1 system, *i.e.* we assume:

1) The input packet flow in the buffer is Poisson of rate λ;
2) Service delays are independent.

In fact the M/G/1 hypothesis is just a matter of simplifying approach. Since we are going to deal with heavy tailed distribution of service times, the consequence on queueing time distribution can be generalized to a much larger class of queueing models. For example, it is not necessary to assume independence between service times or to restrict to Poisson input in order to derive power law queueing distribution (but in this case the coefficients change).

B. Service delay determination

We take the slotted time approach of [3]. IEEE 802.11 CSMA-CA protocol uses a rotating backoff where the nodes have to wait a random number of idle slots between transmission attempts. Let $\mathbf{C}$ be the random variable that expresses the number of busy slots between two consecutive idle slots. Let $p(L)$ be the probability of collision that is experienced by packets.

We take the following assumptions:

1) Times between successive idle slots are independent and i.i.d;
2) Collision events on successive transmissions are independent.

The CSMA protocol assumes that the backoff number is selected in an initial interval $\{1, \ldots, W_{\min}\}$. If a collision occurs the nodes select a new backoff number in an enlarged interval $\{1, \ldots, 2W_{\min}\}$. The backoff interval length is multiplied by two at each collision. The backoff interval length is reset to $W_{\min}$ for the next packet. In practice there is a maximum number of retries after which the packet is discarded in case of permanent failure. The default maximum retry is 7 and can lead to a delay that attains several seconds. Since this delay is larger than the maximum acceptable delay we think of about connection QoS, it does not practically matter to set the maximum number of retries to infinity.

Let $C(z)$ be the probability generating function $\sum_k P(\mathbf{C} = k)z^k$, quantity C being expressed in slot duration. Identity $C(z) = 1$ would mean that $\mathbf{C} = 0$ always, *i.e* the channel is permanently sensed idle.

Let $\beta(z, L, p, k)$ be the probability generating function of the service delay when the packet length is L and when the initial backoff interval is k. From the previous discussion, it comes that it satisfies the following recursion:

$$\begin{aligned}\beta(z, L, p, k) &= \frac{C(z)^{k+1} - C(z)}{C(z) - 1} \frac{z^L}{k} \\ &\quad \times (1 - p + p\beta(z, L, p, 2k)) \ .\end{aligned}$$

The term

$$\frac{C(z)^{k+1} - C(z)}{C(z) - 1} \frac{1}{k} = \frac{1}{k} \sum_{i=1..k} C(z)^i$$

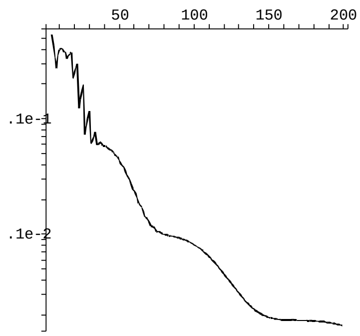

Fig. 1. Coefficients of $\beta(z)$

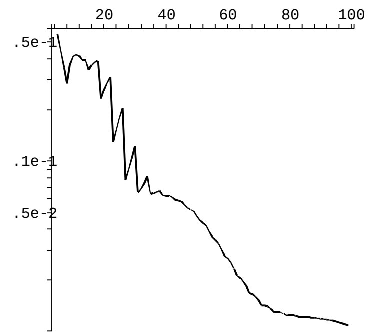

Fig. 2. Coefficients of $w(z)$

corresponds to the delay from the backoff counter decrease, where the backoff timer is selected at random in the interval $\{1, \dots, k\}$.

The term z^L corresponds to the packet transmission time in slots.

Finally, the term $\beta(z, L, p, 2k)$ is obtained from the case where there is a collision (with probability p), hence the procedure is repeated after doubling the backoff interval.

The delay probability generating function is

$$\beta(z) = E[\beta(z, L, p(L), W_{\min})],$$

averaging on packet length L and collision probabilities $p(L)$.

Figure 1 shows the 200 first coefficients of $\beta(z)$ when $C(z) = 0.8z + 0.2z^5$, $L = 4$ and $p = 0.3$.

C. Delays including queueing

We take the formula for slotted M/G/1 in queue delay probability generating function $q(z)$:

$$q(z) = \exp((\beta(z) - 1)\frac{\lambda}{2})\frac{(1 - \lambda\beta'(1))(1 - z)}{1 - z\exp(-(\beta(z) - 1)\lambda)}$$

This needs the provision that $\beta'(1)$ exists. We will see that it implies that $p < \frac{1}{2}$. As well, for the existence of the kth moment of service time we need that $p < 2^{-k}$. If $\lambda \ll 1$ then we could replace by

$$q(z) \approx \frac{(1 - \lambda\beta'(1))}{1 - \frac{z}{1-z}(1 - \beta(z))\lambda}$$

The generating function of the overall delay (queueing + service) of a packet of length L with collision probability p, $w(z, L, p)$ satisfies the identity

$$w(z, L, p) = q(z)\beta(z, L, p, W_{\min}).$$

Figure 2 shows the coefficients of $w(z)$ for $\lambda = 0.02$. Notice that $\beta'(1) = 22.744\cdots$.

III. Asymptotic analysis of delay distribution

Theorem 1: We have the expansion for z around 1:

$$\begin{aligned} w(z) &= 1 + (1 - z)u(1 - z) \\ &\quad + \frac{\lambda(W_{\min}C'(1))^B}{1 - \lambda\beta'(1)}\alpha(\log(1 - z))(1 - z)^{B-1} \\ &\quad + O((1 - z)^B). \end{aligned}$$

where $u(x)$ is an analytic function, $B = -\log_2 p$ assuming that B is not integer, and $\alpha(x)$ is a periodic function of period $\log 2$ with small fluctuation.

Theorem 2: Applying Flajolet-Odlyzko theorems [4] on function $w(z)$, the probability that the delay in a router be greater than T, for T large is

$$\begin{aligned} P(W > T) &= \frac{\lambda(W_{\min}C'(1))^B}{1 - \lambda\beta'(1)}\alpha^*(\log T)T^{1-B} \\ &\quad + O(T^{-B}) \end{aligned}$$

where $\alpha^*(x)$ is also a periodic function of period $\log 2$ with small fluctuation.

Notice that the delay distribution tail decays in power law. As a corollary it turns out that the existence of the kth moment of the delay needs $p < 2^{-k-1}$. The proof is delayed into the appendix.

Remark: The latter assume non integer B, otherwise the conditions are more complex.

A. End-to-end delay

We assume the following

- When travelling on its route, the delay experienced by a packet on a router is independent of the delay experienced on another router.

This assumption makes the problem easier to mathematically handle. However it is not a fundamental assumption for our result since it is known that the sum of two random variables in power law is still in power law whatever the dependence assumptions between them. The power law in the resulting distribution function will be the maximum of the respective power laws of the variables, except that the factor in front of it will depend on the dependence assumptions.

Assuming independence from now, if there are n routers in the route from the source to the destination then the probability generating function of the end-to-end delay is equal to the product $\prod_{i\in\text{route}} w_i(z)$ where $w_i(z)$ is the probability generating function of the delay at router number i and route is a set of router indices.

Still, with Flajolet Odlyzko result [4], if each $w_i(z)$ is of the form

$$1+(z-1)g_i(z)+c_i(z-1)^{B_i-1}+O((z-1)^{B_i}),$$

then the leading term of $P(W(\text{route})>T)$ is

$$\sum_{i\in\text{route}} c_i^* T^{1-B_i},$$

with

$$c_i^*=\frac{c_i}{\Gamma(2-B)}.$$

Keeping only leading terms:

$$P(W(\text{route})>T)\approx c(\text{route})T^{1-B(\text{route})},$$

where $B(\text{route})=\min B_i$ and $c(\text{route})=\sum_{B_j=B} c_j^*$.

An unexpected consequence of the above is that a good choice for the route should not be the shortest path. In the shortest path the lap between two consecutive routers may be too large, leading to too large collision rates and therefore a too low value of $B(\text{route})$. If one takes shorter hops between routers then we will reduce the collision rate and get a larger value of $B(\text{route})$. Of course this would be done in the detriment of a larger number of hops and a larger value of $c(\text{route})$. But since in $c(\text{route})T^{1-B(\text{route})}$, parameter T is supposed to be large the reduction of $T^{1-B(\text{route})}$ would prevail in most cases on $c(\text{route})$ increase.

Interestingly enough, increasing the number of hops and $c(\text{route})$ will in most cases increase the *average* end-to-end delay. Therefore we have the paradoxical case where increasing the average delay actually decreases the overdelay loss ratio. This is due to the fact that we expect the average delay to be much lower than the maximum acceptable delay T. Consequently routing with respect to average delay as it is done in [5] may conflict with the minimization of overdelay ratio.

Conversely the optimal route may be too long since it may have too short hops. In this case the connection may waste too many resources. Instead of choosing the route that minimizes $P(W>T)$ it is probably wiser to seek the shortest route that satisfies the requirement $P(W>T)\leq\epsilon$.

IV. Protocol implementation

A. Collision rates estimates

OLSR uses hellos in order to detect neighbors. A node is a neighbor if and only if the hello collision rate is below a given threshold p_0. Therefore OLSR has a procedure in advanced neighbor sensing option that allows to compute the collision rate (link quality level parameter). It uses the sequence number in order to identify the missing hellos.

However there could be a difficulty in the fact that $p(L)$ may depend strongly on packet length L. One may expect a dependence of the kind $-\log p(L)=aL+b$ where a and b are scalar coefficients. Since the neighbor has no idea of the size of missing hellos, the transmitter should advertize the length distribution of its hellos. Comparing with its received hello distribution the neighbor would be able to determine the coefficients a and b. By default the neighbor assumes $a=0$, *i.e.* all packets have same collision rate regardless of their length.

B. Advertizing Link quality

The OLSR protocol uses topology control optimization. Broadcast traffic is relayed via Multi-Point Relay (MPR) nodes. MPR nodes are elected by their neighbors because they cover their two-hop neighborhood. That way broadcast traffic consumes less resources in order to be forwarded to all destinations. In order to save more on control traffic, nodes have the possibility to advertize a small subset of their neighbor links. The advertized link set can be limited to MPR links, *i.e.* the neighbors that have elected this node as an MPR. In this case the nodes have only a partial knowledge of the network topology. The fact that any given node can compute a shortest path to any arbitrary destination comes from the fact that the node knows its own neighbor list.

For our purpose, it is preferable to use the option full-OLSR, *i.e.* to advertize the whole neighbor set instead of the MPR selector set. To this end a Link Quality Advertizement (LQA) message is used that is broadcasted via MPRs to the whole network. For each link ℓ it advertizes the collision rate p_ℓ, and for the node itself it advertizes the global λ, the global collision rate and the local value of $C'(1)$ (or only the tuple $(p,\frac{\lambda(W_{\min}C'(1))^B}{1-\lambda\beta'(1)})$.

C. Delay based routing

The problem is to find a route that satisfies the end-to-end delay requirement $P(W>T)<\epsilon$ of the new connection. We look to two directions. The first direction consists of finding the route that minimizes $P(W>T)$. The second direction consists of finding the shortest route that satisfies the requirement $P(W>T)\leq\epsilon$.

Finding the optimal route

In general finding the optimal route with respect to a delay distribution is NP hard [2]. But if we stick to the asymptotic expression we can find a polynomial Dijkstra like algorithm.

The idea is to find the route that provides the best asymptotic expansion of the quantity $P(W(\text{route})>T)$ when $T\to\infty$. By best asymptotic expansion we mean the one that provides asymptotically the lowest $P(W(\text{route})>T)$. Since we expect that $P(W(\text{route})>T)$ is asymptotically equivalent to $c(\text{route})T^{1-B(\text{route})}$, the game consists of finding the route which minimizes the sum of the leading terms.

The Dijkstra algorithm is the following. The weight on the links is c^*T^{1-B}. The weight of the route is the sum of the weight of the links: $\sum_{i\in\text{route}} c_i^* T^{1-B_i}$. The optimal route is the route that minimizes this sum. When $T\to\infty$ this would be the route that minimizes the tuple $(B(\text{route}), c(\text{route}))$ with the convention that $(B,c) < (B',c')$ iff either $B > B'$ or $c < c'$ when $B = B'$.

Finding the shortest route

As we have seen before the shortest route that satisfies $P(W > T) \le \epsilon$ is generally preferable to the much longer route that minimizes $P(W > T)$. In the previous section we described a polynomial search which is optimal within a factor $1+O(T^{-1})$. In fact for T sufficiently large the search provides the optimal route. In the present section we want to find the shortest route according to a certain additive metric on links (the hop number) which satisfies a given constraint according to another additive metric (the quantities c^*T^{1-B}).

In general such multi-metric optimization problems are NP-hard. However since the first metric can only take integer values we can easily make it polynomial. We describe the algorithm as follows. We consider a source node A.

Each node B is assigned a vector of integers $(p_0(A,B), p_1(A,B), \ldots, p_n(A,B))$. The value of p_i is the smallest known value according to the second metric of all routes of length i according to the first metric that connect node A to node B.

Clearly we have

$$p_i(A) = \min_{C\in\mathcal{N}(B)} (p_{i-1}(A,C) + p(C,B))$$

where $p(C,B)$ is the value of the second metric on link (C,B) and $\mathcal{N}(B)$ is the neighborhood of node B.

Initializing all vectors to $(\infty,\infty,\ldots,\infty)$ except from the vector of node A itself which is set to $(0,\infty,\ldots,\infty)$, the algorithm will converge in n steps (assuming that all comparisons in parallel in all nodes take one step). Notice that the coefficients $p_i(A,B) = \infty$ for all values of i that are smaller than the distance between A and B (according to the first metric). In order to get the route it suffices to manage a parallel vector that stores the optimal route of length i for all i from 0 to n.

References

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Appendix

At first, we want to prove that

$$\begin{aligned}\beta(1+t, L, p, k) &= 1 + tv(t) + (kC'(1)t)^B\alpha(\log t)\\ &\quad + O(t^{B+1})\end{aligned}$$

where $v()$ is a polynomial.

Additional notations

We fix L and p and set $e^{-\theta} = C(z)$ and denote $j(\theta,k) = \beta(z,k)$. We have

$$j(\theta,k) = \frac{1-e^{-k\theta}}{k\theta} f(\theta)(1-p+pj(\theta,2k))$$

with

$$f(\theta) = e^{\theta}\frac{\theta}{1-e^{-\theta}} z^L.$$

It is clear that

$$\theta = (1-z)C'(1) + O((1-z)^2).$$

We define

$$g(\theta) = \prod_{i\ge 1} \frac{1-e^{-\theta 2^{-i}}}{\theta 2^{-i}}.$$

Thus if $\nu(\theta,k) = g(k\theta)j(\theta,k)$, then

$$\nu(\theta,k) = g(2k\theta)f(\theta)(1-p) + pf(\theta)\nu(\theta,2k)$$

And

$$\nu(\theta,k) = \frac{1-p}{p}\sum_{i\ge 1}(f(\theta)p)^i g(2^i k\theta).$$

Asymptotics analysis

It can be proven that function $g(\theta)$ is analytical and behaves like $1 + O(\theta)$ when $\theta\to 0$ and converges to zero faster than any power law when $\theta\to\infty$.

Let $r_B(\theta)$ be the polynomial of degree $\lfloor B\rfloor$, which is the Taylor expansion of $g(\theta)e^{\theta}$ at $\theta = 0$. Recall that $B = -\log_2 p$.

Let $g_B(\theta) = g(\theta) - r_B(\theta)e^{-\theta}$. Clearly $g_B(\theta) = O(\theta^{\lceil B\rceil})$ when $\theta\to 0$.

We have

$$\nu(\theta,k) = u_B(\theta) + \frac{1-p}{p}\sum_{i\ge 1}(f(\theta)p)^i g_B(2^i k\theta)$$

with

$$u_B(\theta) = \frac{1-p}{p}\sum_{i\ge 1}(f(\theta)p)^i r_B(2^i k\theta)e^{-2^i k\theta}.$$

Clearly $u_B(\theta)$ is an analytical function with $u_B(\theta) = 1 + O(\theta)$.

Let

$$\nu_B(\theta,k) = \frac{1-p}{p}\sum_{i\ge 1}(f(\theta)p)^i g_B(2^i k\theta).$$

We will show that $\mu_B(\theta,k) = \theta^{-B}\nu_B(\theta,k)$ is bounded when $\theta\to 0$. Let $h_B(\theta) = \theta^{-B} g_B(\theta)$.

We have

$$\mu_B(\theta,k) = \frac{1-p}{p}\sum_{i\ge 1}(f(\theta))^i h_B(2^i k\theta)k^B.$$

Since $f(\theta) = 1 + O(\theta)$, when $\theta \to 0$, we have $\mu_B(\theta, k)$ which converges to $\alpha(\log\theta) = \sum_i h_B(2^i k\theta)k^B$, the sum being on all integers i, including the negative integers. The sum converges because $h_B(\theta) = O(\theta^\epsilon)$ with $\epsilon = \lceil B \rceil - B$ and $h_B(\theta)$ decays faster than any power law. Notice that function $\alpha(x)$ is periodic of period $\log 2$.

Therefore $j(\theta, k) = \frac{\nu(\theta,k)}{g(k\theta)}$ has asymptotic expansion

$$\frac{u_B(\theta)}{g(k\theta)} + \alpha(\log\theta)\theta^B + O(\theta^{B+\epsilon}).$$

Since $\frac{u_B(\theta)}{g(k\theta)}$ is analytical and equal to 1 at $\theta = 0$, then $j(\theta, k)$ fits Flajolet-Odlyzko asymptotic conditions.

Now, using the expansions for $\beta(z)$ and $q(z)$ around $z = 1$ and the formula $w(z) = q(z)\beta(z)$, we have the expansion for $w(z)$:

$$\begin{aligned} w(z) &= 1 + (1-z)u(1-z) \\ &\quad + \frac{\lambda(W_{\min}C'(1))^B}{1-\lambda\beta'(1)}\alpha(\log(1-z))(1-z)^{B-1} \\ &\quad + O((1-z)^B). \end{aligned}$$

By writing the function $\alpha(\log(1-z))$ as a Fourier series:

$$\alpha(\log(1-z)) = \sum_n \alpha_n (1-z)^{\frac{2in\pi}{\log 2}},$$

and applying Flajolet-Odlyzko theorems, we have

$$\begin{aligned} P(W > T) &= \frac{\lambda(W_{\min}C'(1))^B}{1-\lambda\beta'(1)}\alpha^*(\log T)T^{1-B} \\ &\quad + O(T^{-B}), \end{aligned}$$

where α^* is periodic in $\log T$, of period $\log 2$:

$$\alpha^*(\log(T)) = \sum_n \frac{\alpha_n}{\Gamma(2 - B - \frac{2in\pi}{\log 2})} T^{\frac{2in\pi}{\log 2}}.$$